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A Rational Expectations Model of Agricultural Supply

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This work suggests a dynamic linear rational expectations modeling strategy for using time-series observations to study the impact of product prices on agricultural production and land allocation. The farmer's decision consists of specifying each period the portion of his land to be devoted to the growing of alternative crops. The dynamic element in the farmer's problem comes from the technology. A quadratic production technology is used. The farmer's optimal land allocation decision rules are derived as linear functions of past land allocations, expectations of future product prices, and other exogenous variables; and they can be interpreted as an optimal crop rotation. Assuming rational expectations and the exogeneity of prices for a small open economy, one obtains closed-form linear regression equations representing decision rules and stochastic processes. These are then estimated and tested using maximum likelihood methods and aggregate data from the Egyptian agriculture.

I. Introduction

Using annual data, economists have suggested different theoretical and empirical methods to evaluate farmers' responses to changes in crop prices. The existence of persistent patterns of serial and cross correlations between land allocations, production, and prices has been observed and debated in the economic literature for many years.

This paper is based on my 1981 Ph.D. dissertation at the University of Minnesota. I am grateful to Anne O. Krueger, Lars Hansen, Thomas Sargent, Christopher Sims, Vernon Ruttan, Rao Aiyagari, Martin Eichenbaum, Wallace Huffman, Dan Peled, T. N. Srinivasan, and Ken Wolpin for useful comments on previous drafts of this paper. Comments by the referees and editor of this journal were most useful.
The fact that output selling price is not known when input decisions are made and the resulting need for farmers to make decisions based on expectations of future prices have been suggested as the main reasons for the cyclical movements of output. Early single-equation estimates, with current output as a function of only one past price, showed small linkage between prices and output. Later, Nerlove (1956, 1958) showed that the static supply equation, with the assumptions of "partial adjustments" in actual output (land) and adaptive expectations of the output price, gave rise to a single distributed lag model that could explain much of the supply response to output price changes. Askari and Cummings (1976) report more than 600 estimates of different versions of Nerlove's model for many crops and countries. Muth (1961) criticized Nerlove's model in the light of his rational expectations model.\footnote{See Behrman (1968) for a detailed discussion of the issues and complete country work that follows the Nerlovian model. More recently, Nerlove (1979) critically reviewed the traditional supply response model in the light of recent developments in economic time-series models; e.g., Muth (1961); Lucas (1975); and Nerlove, Grether, and Carvalho (1979). In my dissertation (Eckstein 1981b) there is some further discussion of the subject.}

In this study, an empirical model of agricultural supply is derived from a dynamic and stochastic framework where farmers are assumed to maximize the expected present value of profit by choice of land allocations subject to a dynamic and stochastic technology as well as uncertain price movements.\footnote{This approach follows Sargent (1979a, 1981) and is consistent with Schultz's (1978, p. 4) view on farmers' behavior.} The analysis focuses on the dynamics of crop areas and their joint movement with crop prices. In this context, it is straightforward to show that rational farmers are unlikely to interpret price fluctuations that are serially uncorrelated as signaling a permanent alteration in the incentives confronting them. Furthermore, any permanent or transitory change in taxes, subsidies, or tariffs affects the dynamic response of the cropped area, such that the structural form of the land allocation equation varies directly with policy rules. Consequently, predictions with respect to changes in policy require complete identification of the structural parameters that govern the production and the price processes (Marschak 1953, Lucas 1976).

Understanding the consequences of price movements on agricultural production is of particular importance in less developed countries. Therefore, the model is implemented using aggregate agricultural data from Egypt, 1913–69. The dynamic statistical properties of the data were initially analyzed by the estimation of vector autoregressions (VAR) that included cotton and wheat cropped areas, yields, and prices. The results showed (1) that the percentage of forecast
error produced by a shock to prices accounted for 15–50 percent of the total variance in land allocations and yields. Hence, price movements have a significant effect on farmers’ decisions. (2) A shock in any variable causes cotton and wheat areas to respond in opposite ways and to fluctuate frequently as they converge to their mean. Figure 1 shows this result for an innovation in the cotton price over the wheat price in a particular system. It is of interest now to find out whether a rational expectations model that accounts for some technological aspects of agricultural production can interpret well the properties of the Egyptian data.

The remainder of the paper is organized as follows: In Section II, I construct a model where the dynamics in the production process are

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3 The methodology for estimating and interpreting VAR models was developed by Sims (1980), who used it to analyze macroeconomic questions. T. Doan and R. Litterman’s package of Regression Analysis of Time Series (RATS), version 4.1, has been used for computations. The results from estimating several unrestricted VARs have preceded the formulation of the models. I estimated several different VARs and the results turned out almost the same. Detailed information exists in my dissertation (Eckstein 1981). Figure 1 is taken from a system of cotton and wheat areas and yields and the relative price (cotton price over wheat price).
II. A Dynamic Land Allocation Model

In this section I analyze the stochastic dynamic optimization problem of a farmer endowed with land that can be allocated between two crops (e.g., cotton and wheat). The model shows that if the cultivation of at least one crop (e.g., cotton) results in deterioration of land productivity, the optimization yields a dynamic land allocation process that can be estimated using aggregate time-series data.

The role of adjustment costs in affecting input decisions and output supply has been analyzed extensively in the economics literature (e.g., Lucas 1967; Gould 1968; Sargent 1979b) and to some extent also in the agricultural economics literature (e.g., Nerlove 1958, 1972; Nerlove et al. 1979). Adjustment costs in annual crop production could be justified when land preparation of a plot has crop-specific requirements. When land is continuously cultivated, the issue of substitution and complementarity effects in the production of alternative crops becomes important. The depletion of nitrogen from the soil is an important direct constraint on the development of land fertility and the production of all crops. Furthermore, monoculture causes an accumulation of crop-specific insects and worms, which have an important indirect effect on the actual crop yield from the land. Hence, the current productivity of land for a given crop depends on the cropping history of a plot of land.\footnote{Crop rotation is the well-known method to prevent the direct and the indirect deterioration in land productivity under continuous cultivation. Fertilizer and pesticides are the main inputs that directly control land productivity by building up the content of the soil and eliminating insects and worms. For the importance of crop rotation and land fertility deterioration in the United States, the reader is referred to Nerlove (1958, pp. 122-35) and the many publications on this subject by the U.S. Department of Agriculture. In Egypt, the case study here, Hansen and Marzouk (1965) and Hansen and Nashashibi (1975) emphasized the crucial effect of nitrate depletion in cotton production on the acreage allocation of all crops.}

To develop a simple model that contains these technological characteristics, consider the following definitions:

- $X_i(t)$ is the production of crop $i$ at time $t$;
- $P_i(t)$ is the price that farmers receive for the production of crop $i$ at time $t$.\footnote{Crop rotation is the well-known method to prevent the direct and the indirect deterioration in land productivity under continuous cultivation. Fertilizer and pesticides are the main inputs that directly control land productivity by building up the content of the soil and eliminating insects and worms. For the importance of crop rotation and land fertility deterioration in the United States, the reader is referred to Nerlove (1958, pp. 122-35) and the many publications on this subject by the U.S. Department of Agriculture. In Egypt, the case study here, Hansen and Marzouk (1965) and Hansen and Nashashibi (1975) emphasized the crucial effect of nitrate depletion in cotton production on the acreage allocation of all crops.}
$A_\tau$ is the land allocated at time \( t - 1 \) for the production of
crop \( i \) at time \( t \);
$\lambda$ is the total available cultivated land at time \( t \);
$0 < \beta < 1$ is the objective discount factor;
$\alpha_\tau$ is the shock to production of crop \( i \) at time \( t \);
$E$ is the mathematical expectation operator, where $E(X) = E(X|\Omega_t)$ and \( \Omega_t \) is the information set available for farmers at time \( t \); and
$L$ is the lag operator defined by the property $L^AX_t = X_{t-k}$.

Suppose that only crop 1 (cotton) is subject to dynamic constraints in
production; therefore I can write

$$X_{1t} = F_1(A_{1t}, a_{1t}, A_{1t-1}, A_{1t-2}, \ldots; \mathbf{K}, \lambda),$$  \hspace{1cm} (1)

where $F_1 > 0$, $F_2 > 0$, $F_j \leq 0$ for all $j \geq 3$, $F_1 \leq 0$, $\mathbf{K}$ is a vector
of other (fixed) inputs, and $\lambda$ is the size of a given plot of cultivated
land. The negative sign of the derivative of the production of crop 1
with respect to lagged allocations of land for the same crop ($F_j$, $j \geq 3$)
is due to the assumption of deterioration in productivity with respect to
an increase in the past intensity of use of the same plot with the same
crop. The production function of crop 2 (wheat) is given simply by

$$X_{2t} = F_2(A_{2t}, a_{2t}, \mathbf{K}, \lambda),$$  \hspace{1cm} (2)

where $F_2 > 0$, $F_3 > 0$, and $F_4 \leq 0$. The representative farmer is
assumed to maximize his expected discounted profit in terms of the
price of crop 1 (cotton) by choosing a contingency plan at each period
\( t \) for allocating his given area ($\lambda$) for the time \( t + 1 \) production of the
two crops. Hence, the farmer's objective is to maximize

$$E^{-1} \lim_{\lambda \to \infty} \sum_{t=1}^{\infty} \beta^t \left( X_{1t} + \frac{P_{2t}}{P_{1t}} X_{2t} \right),$$  \hspace{1cm} (3)

subject to the land constraint

$$A_{1t} + A_{2t} = \lambda$$  \hspace{1cm} (4)

and the production functions (1) and (2). The optimization is subject to
the given levels of $A_{1t}$, $j < 0$, the well-defined joint distribution of

\footnote{The partial derivative of $F_j$ ($i = 1, 2$) with respect to the variable in the \( j \)th position
in the function is $F_j$. The asymmetry between cotton (1) and wheat (2) production
technology is motivated by the Egyptian case, where cotton productivity is severely
affected by past cultivation patterns but wheat production is not.
\footnote{The choice of the price in the numerator affects the solution to the optimization
problem. Here I divide by $P_1$, in order to preserve the quadratic form of the model.}
\([\{P_{2t}/P_{1t}, a_{1t}, a_{2i}\}_{t=0}^{\infty}\], and the assumed information set at the time the decision for period \(t\) is made, i.e.,

\[
\Omega_{t-1} = \left\{ A_{1t-1}, A_{1t-2}, \ldots, a_{1t-1}, a_{1t-2}, \ldots, \frac{P_{2t-1}}{P_{1t-1}}, \frac{P_{2t-2}}{P_{1t-1}}, \ldots, S_{t-1}, S_{t-2}, \ldots \right\}.
\]

Given that certain additional regularity conditions are met, the optimization problem (3) has a unique solution for the land allocation for crop 1 at time \(t\) that is a function of the information set \(\Omega_{t-1}\). Since this paper is concerned with the empirical implications of the model for aggregate data, I seek an analytical solution to problem (3). I suggest a linear-quadratic approximation for the production function above and the stochastic processes of the uncontrolled variables, and I assume that both crops can be produced on the same plot at the same period (4).\(^7\) In particular, let (1) be specified as

\[
X_{1t} = \left( f_1 + a_{1t} - \frac{g_1}{2} A_{1t} \right) A_{1t} + d_1 \left( 1 - \frac{A_{1t-1}}{\bar{A}} \right) A_{1t-1} \tag{5}
\]

where \(f_1, g_1,\) and \(d_1\) are positive scalars. The first term represents a standard quadratic function. The second term is meant to approximate the deterioration in land productivity. For \(d_1 > 0\), the particular approximation suggests that if the summation of the fractions of land from last and current periods is greater than one, then the current average productivity of land is reduced. Furthermore, if the summation of \(A_{1t}/\bar{A}\) and \(A_{1t-1}/\bar{A}\) is less than one, the current cultivation of crop 1 is assumed to be on land that has been used for crop 1 for only the current year. Hence, the average productivity is increased. If the sum of \(A_{1t}/\bar{A}\) and \(A_{1t-1}/\bar{A}\) is equal to one, there is no linkage between the current average productivity of land and past cultivations. Hence, only if it turns out that \(A_{1t}/\bar{A} = \frac{1}{2}\) for all \(t \geq 0\) would the farmer's problem seem to be static. Observe that if \(d_1 < 0\), the production function (5) implies that \(F^{\frac{1}{2}} > 0\); i.e., past cultivation with the same crop increases current productivity. This can be explained as arising from “high costs” of crop-specific plot preparations, which hold for several seasons.

\(^7\) Alternatively one may assume that each plot is rigidly defined and that it can be used either for crop 1 or for crop 2. In this case the land allocation decision is a discrete choice problem, which is tractable and is not an attractive description of the country aggregate or (even) the farm data. However, in the discrete case the production function (5) is intuitively very appealing, and if one has micro data on the history of plots one may use the above approximation to the farmer problem in order to derive a probit model.
The production function of crop 2 (wheat) is assumed to be linear:

\[ X_{2t} = (f_2 + a_{2t})A_{2t}. \]  (6)

The specification of the model above is of particular interest since it enables one to (i) derive an analytical solution to the farmer's problem (3); (ii) emphasize the dynamic aspects of the production function of the main crop (cotton); (iii) keep the model close to the dynamic supply model of Nerlove (1958); and (iv) show what sort of restrictions one should impose on the theoretical farmer's problem in order to justify explicitly the linear time-series econometric models that are widely used. The production functions (5) and (6) are very simple approximations, but the Egyptian aggregate data did not a priori reject the specification of the cotton production function (5). ¹ In general, an extended version of (5) that includes more lagged area and other inputs (e.g., labor and fertilizer) could be specified (see the App.), but only numerical solutions to the farmer's problem are feasible for much more complicated models.

If we substitute (4)–(6) into (3), the farmer's problem becomes:

maximize

\[ J = E_{-1} \lim_{N \to \infty} \sum_{t=0}^{N} \beta^t \left[ (f_1 + a_{1t})A_{1t} - \frac{g_1}{2} A_{1t}^2 \right. \]

\[ + \frac{d_1}{\bar{A}} (\bar{A} - A_{1t-1} - A_{1t})A_{1t} - R_i A_{1t} + R_i \bar{A} \],

(7)

by choice of \( A_{10}, A_{11}, A_{12}, \ldots \), where \( R_i = (1/P_i) \left[ P_2(f_2 + a_{2t}) \right] \) is the "real" shadow price for crop 1 land allocations. The optimization is subject to a given level of \( A_{1t-1} \) and a given law of motion for the stochastic processes of \( a_{1t}, R_i, \) and \( S_i \), i.e.,

\[ \delta(L)Z_t = U_t, \]  (8)

where \( Z_t = (a_{1t}, R_i, S_i) \); and \( \delta(L) = I - \delta_1 L - \delta_2 L^2 - \ldots - \delta_k L^k \), where \( S_i \) is a vector of \( n - 2 \) exogenous variables at time \( t \) (such as taxes, tariffs, and other variables that contain information on \( P_i \)'s and \( a_{1t} \)'s) and \( \delta_j \) is an \( n \times n \) matrix for \( j = 1, \ldots, k \). Further, \( U_t \) is an \( n \times 1 \) vector where \( E(U_t|\Omega_{t-1}) = 0 \) and \( E(U_t^\prime U_t) = \Sigma_t \) and where \( \Sigma_t \) is a positive semidefinite matrix. The vector stochastic process (8) is as-

¹ In Eckstein (1981b) I estimated it directly using aggregate data from Egypt, 1895–1975. (Data on prices are available only from 1913 and have been fixed by the government since 1968.) I divided (5) by \( A_{1t} \) to get a linear yield equation with a first-order serially correlated error (\( a_{1t} \)—see Sec. III). The results from the two-step efficient method of estimation were that (5) is significantly quadratic (i.e., \( [(g_1/2) + (d_1/\bar{A})] > 0 \), the serial correlation coefficient of \( a_{1t} \) is a significant positive fraction, and \( d_1 \) is an insignificant positive number. Furthermore, a likelihood ratio test for a structural change in (5) after World War II was rejected at the 5 percent significance level.
sumed to be of mean exponential order less than $1/\sqrt{\beta}$, so that a constant and a linear trend can be part of the vector $S_t$. The variables in the vector $Z_t$ $(n \times 1)$ are viewed as being unaffected by the farmer’s decisions. Thus, for example, prices are assumed to be exogenously given to the representative farmer.

Following Sargent (1979b), the Euler equations of the dynamic problem (7) can be written as

$$A_{1t} = \lambda_1 A_{1t-1} - \frac{\lambda_1 \Delta}{d_1} \sum_{j=0}^{\infty} \lambda_1^{j+1} \left[ \beta \lambda_1 / (\beta_1 + d_1 + E_{t-1}(a_{1t+j}) - E_{t-1}(R_{t+j})) \right]$$

(9)

for all $t = 0, 1, 2, \ldots$, where $\lambda_1$ is a function of $g_t, d_t, \bar{A}$, and $\beta_1 < -1 < \lambda_1 < 0$ if $d_1 > 0$, and $0 < \lambda_1 < 1$ if $d_1 < 0$.

For any arbitrary set of expectations, (9) implies that

$$\frac{\partial A_{1t}}{\partial E_{t-1}(R_{t})} < 0 \quad (10a)$$

and

$$\frac{\partial A_{1t}}{\partial E_{t-1}(R_{t+1})} > 0 \quad (10b)$$

for $d_1 > 0$. Equation (10a) is the standard result that more land is allocated to crop 1 as the expected price of the season goes up. However, (10b) implies that if farmers expect that in the following year the price of crop 2 relative to the price of crop 1 is going to decrease, they will decrease the quantity of current land allocated to crop 1. This result could be viewed as a rationale for the “cobweb phenomenon,” that is, that the inherent dynamics in the production function cause cotton and wheat production to oscillate in response to changes in the (rational) expected price in the future. Dynamic models with adjustment costs in land allocations ($d_1 < 0$) imply the same result for (10a) but the opposite sign for (10b).

The assumption of rational expectations in this model implies that farmers maximize (7) subject to the true stochastic process of the exogenous variables (8). Using Hansen and Sargent’s (1980) predic-

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1 In Eckstein (1981a) I prove the equivalence of problem (7) where $d_1 < 0$ with the standard adjustment-costs problem discussed in the literature. The Neumannian supply response model uses the adjustment-costs argument to justify adjustment in actual area vis-à-vis desired land allocations. If I include more than a 1-year deterioration effect in (5), the number of lags of land allocations in (9) will be equal to the number of years in the cumulative dynamic factor in the production function (see Hansen and Sargent 1981). If (5) is linear—i.e., $(g_t/2) + (d_t/A) = 0$—then the optimization problem (7) is not concave in $A_t$, and either there is a solution where $A_{1t} = A \leq \bar{A}$ for all $t$, or $A_{1t} = \bar{A}$ for some $t$ and $A_{1t} = 0$ otherwise ("bang-bang"). In my (1981b) dissertation I show that with constant prices both cases are possible, depending on the prices of wheat and cotton.
tion method, we can write the optimal decision rule as a function of variables in the farmer's information set at time \( t \); i.e.,

\[
A_{1t} = \lambda_1 A_{t-1} + \gamma + \mu_1(L)A_{t-1} + \mu_2(L)R_{t-1} + \mu_3(L)S_{1t-1} \\
+ \mu_4(L)S_{2t-1} + \ldots + \mu_n(L)S_{n-2t-1},
\]

(11)

where

\[
\gamma = \frac{\lambda_1 \tilde{X}(f_t + d_t)}{d_t \beta(1 - \beta \lambda_1)}
\]

and

\[
\mu_i(L) = \mu_{i(0)} + \mu_{i1}L + \ldots + \mu_{ij}L^j
\]

for all \( i = 1, 2, \ldots, n \) and \( j \leq k \). Equation (11) is an exact closed-form analytical solution for the farmer's optimal land allocation decision rule at time \( t \). Observe that \( \mu_i \)'s coefficients are some nonlinear function of \( \lambda_1, \beta, d_t \), and \( \delta_i \)'s coefficients, which expresses the restriction imposed across the decision rule and the parameters of the stochastic processes for variables in \( Z_t \).

The certainty equivalence property of the model implies that under certainty the optimal decision rule (9) depends on the actual values (perfect foresight) of \( a_{t+i} \) and \( R_{i+j} (j = 0, 1, \ldots) \) and that under uncertainty it depends on their conditional expected values but not on other moments of the distributions of the random variables. Imposing the uncertainty on the farmer's problem is of interest here for two reasons. First, theoretically, past exogenous variables are correlated with current decisions only because of the information they contain about their future realizations (as in [11]). Changes in the distributions of the exogenous variables would alter the correlations through the cross-equations restrictions. Hence, predictions with respect to a permanent change in relative prices require a complete identification of the model's parameters, even though prices are exogenous (Lucas 1976). Second, under uncertainty equation (11) has the standard econometric form of distributed lag models that are estimable using standard nonlinear methods. On the other hand, under certainty equation (9) contains all future exogenous variables, and it

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19 Hansen and Sargent's (1980) optimal projection for (9) given (8) and the information set \( \Omega_{t-1} \), is

\[
\sum_{j=0}^{\infty} \lambda^j E_{t-1}(g_{t+j}) = \mathbf{v} \cdot \left[ \lambda^{-1} \delta^{-1}(\lambda) \left[ I + \sum_{j=1}^{\infty} \left( \sum_{\gamma=1}^{\infty} \lambda^j \delta_j \right) L^j \right] - \lambda^{-1} I \right] \cdot Z_{t-1},
\]

where \( \mathbf{v} = [1, 1, 0, 0, \ldots, 0] \) is a row vector with ones in the first two positions and zeros in the next \( n - 2 \) positions, and where \( \mathbf{v} \cdot Z_t = c_t = (a_t - R_t)I, \lambda = \lambda_1 \beta, \) and \( I \) is an \( n \times n \) identity matrix.
cannot provide an integrated aggregate error term for estimation (see Sec. III). Therefore, it is not a tractable estimable equation.

In order to analyze (11) let us consider the following case. The shocks to production ($a_t$'s) and the price ($R$) are serially uncorrelated and are independent of variables that are in the information set, $a_t$, has zero mean and $R_t$ has a positive mean. Equation (11) can be written as

$$A_{1t} = \lambda_1 A_{1t-1} + \frac{\lambda_1 \bar{A}}{d_1 (1 - \lambda_1)} \cdot \text{(mean of } R), \quad (12a)$$

and the mean of $A_{1t}, A^*_1$, is

$$A^*_1 = \frac{\gamma}{1 - \lambda_1} + \frac{\lambda_1 \bar{A}}{d_1 (1 - \lambda_1 \beta)(1 - \lambda_1)} \cdot \text{(mean of } R). \quad (12b)$$

For the relevant domain of $d_1$, we obtain $\partial A_{1t}/\partial d_1 < 0$ and $\partial A^*_1/\partial d_1 < 0$. Thus, increasing the rate of land deterioration decreases the area allocated to crop 1. Equations (12a) and (12b) show that farmers would not interpret price fluctuations and shocks to production that are serially uncorrelated as signaling a permanent alteration in the incentives confronting them. Now consider the experiment of a one-and-for-all increase in the mean of the relative price, $R$. Using equations (12a) and (12b) the immediate response for $A_{1t}$ is a decrease below the (lower) new level of $A^*_1$ and, by oscillating fluctuations, convergence toward the new mean of $A_{1t}$. Hence, the short-run effect is greater than the long-run, and the cobweb phenomenon is, in this model, an optimal response to a shock in relative prices and has nothing to do with rational price expectations. At the same time, the model predicts the same response to a shock as the VAR (fig. 1) using Egyptian data on cotton and wheat. Observe that the qualitative implication of this experiment holds in the general case as well and can illustrate the effects of deterministic tax (subsidy) policies on the movements of land allocations. It is straightforward to see that in the case of adjustment costs ($\lambda_1 > 0$ and $d_1 < 0$), the signs of both the immediate and the long-run effects of the experiment are retained but the magnitude of both increases. However, the short-run effect is lower than the long-run (see Nerlove 1958) and the convergence toward the mean is a downward smooth path rather than oscillations as in the case where $d_1 > 0$. In general, the structure of the stochastic process of the relative price has an important effect on the predicted movements of land allocations due to changes in prices or other variables that affect prices. This includes the magnitude of the difference between the immediate response (short run) and the average change (long run) in land allocations due to changes in prices.

What are the effects of fertilizer, labor, and pesticides on the land
allocation decision rule? In general, if the production function of crop I is separable between land and any other inputs, the decision rule (11) stays the same. Theoretically, one can specify a production function that exhibits a complicated interaction between factors of production and that includes both static and dynamic elements. Hansen and Sargent (1981) discuss methods for solving these types of models.\textsuperscript{11}

Estimating the underlying parameters of the model is crucial in understanding supply responses and the land allocation decision process. Equation (11) is almost a regression equation. If I do not observe some of the variables that are part of the farmer's information set, I can construct an error term for (11) such that it has the properties of a regression equation. The reduced form of this equation is observationally equivalent to the traditional supply response model of Nerlove (1958), since both models can be specified such that they have identical reduced-form equations. However, the model in this paper has a completely different interpretation of the observed pattern of serial and cross correlations between crop areas and crop prices.\textsuperscript{12}

The traditional supply response model does not investigate jointly the dynamics of the production process and the dynamics of the actual crop prices that farmers observe. The model here emphasizes the important role of the dynamic structure of the production technology, the information farmers have at the time inputs are committed to production, and the way relative prices are moving over time, in the determination of farmers' response to changes in incentives.

III. Estimating the Dynamic Land Allocation Model

The reason for estimating the model is to determine whether a dynamic rational expectations model is reliable for prediction and capable of interpreting the observed serial and cross correlations between crop areas and prices. In the discussion above I showed that the model can interpret oscillating fluctuations in land allocations, such as those that arise in a VAR (fig. 1) using Egyptian data, as a rational response to shocks in prices and productivity. As such, there is reason to believe that the restrictions imposed on the data by the model will not be rejected by a careful use of statistical methods.

Here I present results from a maximum likelihood estimator of a

\textsuperscript{11} The Appendix illustrates how the interaction between inputs may affect the dynamic properties of the model.

\textsuperscript{12} See Sargent (1976) for a similar result with respect to macroeconomic models. In Eckstein (1981a) I show how a simple Nerlovian model can fit perfectly well data generated from a specific case of eq. (11).
simple bivariate specification of the land allocation model using Egyptian annual data on cotton area, cotton lint price, wheat price, and wheat yield from 1913 to 1969. Cotton and wheat are the main crops in the Egyptian economy, and soil deterioration and insect accumulation in soils under continuous cotton production are the main reasons for crop rotation in Egyptian agriculture. During the relevant period both cotton and wheat were internationally traded in large quantities. Hence, it is reasonable to assume that their prices are determined in the world markets and are not Granger caused (Granger 1969) by Egyptian production. The average area and yield of cotton and wheat stayed almost the same over the entire period, but the annual quantities show large fluctuations over time. Given the importance of agriculture in the poor Egyptian economy, the characteristics of the main crops, and the fact that most recent studies on the dynamics of agricultural supply were of crops in less developed countries (see Askari and Cummings 1976), it is particularly interesting to test this model with the fairly good Egyptian data.\(^{13}\)

As in traditional agricultural supply response models, the two variables here are the cotton crop area \((A_{1t})\) and the relative price \((R_t)\). The decision rule (11) and the vector stochastic process (8) are the joint equations that are estimated here. The unrestricted (reduced-form) system of two equations \((A_{1t}, R_t)\) yields a VAR of a lag length that is determined by the stochastic process for \(R_t\) and \(a_{1t}\). Therefore, an unrestricted VAR can serve as a statistically maintained alternative to any restrictions that the theory imposes on the data. Likelihood ratio tests of lag length for the complete unrestricted (symmetric lags) rejected two versus four lags (marginal confidence level = .92) but did not reject three versus four lags (marginal confidence level = .44).

Two types of identifying restrictions exist here: (i) zero restrictions, which amount to the assumption that \(A_{1t}\) does not Granger cause \(R_t\), and (ii) cross-equation restrictions due to the rational expectations assumption, given that \(i\) is correct (see Sargent 1981).

\(^{13}\) Almost identical data were used by Hansen and Nashashibi (1974, 1975) and are available also in Eckstein (1981b). The prices are ex-farm prices in Egypt and the areas and yield are national aggregates. The sources of the data are Egypt Ministry of Finance, Annuaire Statistique, 1909-60, and National Bank of Egypt, Economic Bulletin, 1948-78. Detailed discussions of the Egyptian agricultural sector are available in Hansen and Marzouk (1963), Owen (1969), Hansen and Nashashibi (1975), and Eckstein (1981b). Note that cotton is the main crop in production, and both the lint and the seeds have been the main sources of export earnings for many years (since 1880). Wheat is second to cotton in production, its growing period overlaps cotton's, and it is a part of the crop rotation system that Egyptian farmers follow. Furthermore, wheat became an important imported commodity, and substitution between wheat and cotton in production has a direct effect on the trade balance.
In order to preserve the simplicity of the model and the three-lag VAR supported by the tests, I assume that $R_t$ and the shocks to productivity ($a_{1t}$) have the following autoregressive processes:

\[
\begin{align*}
R_{t} &= \alpha_0 + \alpha_1 R_{t-1} + \alpha_2 R_{t-2} + u^R_t \\
A_{1t} &= \rho A_{1t-1} + \epsilon_t
\end{align*}
\]  

(13)

where it is assumed that $|\rho| < 1$ and the roots of $|1 - \alpha_1 z - \alpha_2 z^2| = 0$ are outside the unit circle.

Using the farmer’s land allocation decision rule (11), and given that $a_{1t}$ is not observed, I can write the VAR for $A_{1t}$ and $R_t$ as

\[
\begin{align*}
\begin{bmatrix}
A_{1t} \\
R_t
\end{bmatrix}
&= \begin{bmatrix}
\mu_0 \\
\mu_1
\end{bmatrix} + \begin{bmatrix}
w_1 \\
w_2
\end{bmatrix} W A R + \begin{bmatrix}
\rho + \lambda_1 & \mu_1 \\
0 & \alpha_1
\end{bmatrix} \begin{bmatrix}
A_{1t-1} \\
R_{t-1}
\end{bmatrix} \\
&+ \begin{bmatrix}
-\rho \lambda_1 & \mu_2 - \rho \mu_1 \\
0 & \alpha_2
\end{bmatrix} \begin{bmatrix}
A_{1t-2} \\
R_{t-2}
\end{bmatrix} + \begin{bmatrix}
0 & -\rho \mu_2 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
A_{1t-3} \\
R_{t-3}
\end{bmatrix} + \begin{bmatrix}
\epsilon_{1t} \\
\epsilon_{2t}
\end{bmatrix},
\end{align*}
\]  

(14)

where $WAR$ represents dummies for the Second World War (1941–45), $\mu_0$ contains several deterministic (time-independent) elements from the decision rule (11) that cannot be identified separately, $\epsilon_{1t} = \mu_3 u^a_{t-1}$, and $\epsilon_{2t} = u^R_t$; $\mu_1$, $\mu_2$, and $\mu_3$, defined in (15), contain the restrictions across the equations in (14) and represent the implications of the rational expectations hypothesis:

\[
\begin{align*}
\mu_1 &= \frac{\lambda_1}{d} \left( \frac{\alpha_1 + \alpha_2 \lambda}{1 - \alpha_1 \lambda - \alpha_2 \lambda} \right) \\
\mu_2 &= \frac{\lambda_1}{d} \left( \frac{\alpha_2}{1 - \alpha_1 \lambda - \alpha_2 \lambda^2} \right) \\
\mu_3 &= \frac{\lambda_1}{d} \left( \frac{-\rho}{1 - \rho \lambda} \right),
\end{align*}
\]  

(15)

where $d = d/\bar{A}$. Here $\epsilon_t = (\epsilon_{1t}, \epsilon_{2t})'$ is the vector of innovations that is assumed to have a bivariate normal distribution with $E(\epsilon_t \epsilon_t') = \Sigma$.

---

14 $R_t$ = wheat price × wheat yield × (1/cotton lint price)

15 The Second World War caused exogenous limitations on trade and altered the level of wheat and cotton production. To capture this effect a dummy was introduced to the empirical specifications.

16 See n. 10 for the method.

17 Equation (13) is a particular specification for eq. (8). The lag order in the $R_t$ process is supported by estimating univariate autoregressive processes. Strategies for estima-
Hence, estimators of the vector of free parameters \( \theta = \{ \lambda_1, d, \rho, \alpha_1, \alpha_2, \alpha_3, \mu_1, w_1, w_2 \} \) is obtained by maximizing the likelihood function with respect to \( \theta \). Let \( \mathbf{l}_t = (l_{1,t}, l_{2,t})' \) be the sample residual vector for a given value of the parameter vector \( \theta \). Then the concentrated log-likelihood function of the sample of observations on the residuals over \( t = 1, \ldots, T \) is (see Sargent 1978)

\[
L(\theta) = -T[\log (2\Pi) + 1 - \log T] - \frac{1}{2}T \log \left| \sum_{t=1}^{T} \mathbf{l}_t(\theta)\mathbf{l}_t(\theta)' \right|. \tag{16}
\]

Equation (16) is maximized with respect to \( \theta \), where \( \mathbf{l}_t(\theta) \) is defined by (14) and (15) for each observation.\(^{18}\) Observe that (14) has 11 nonzero regressors while the vector \( \theta \) has only nine free parameters. Hence, there are two overidentifying restrictions that are imposed by theory as implied by (15). These restrictions as well as the a priori zero restrictions are tested using conventional likelihood ratio tests.

The estimated parameters (see table 1) of the model satisfy the restrictions that the model imposed on the farmer's problem; that is, \( |\lambda_1| < 1, |\rho| < 1 \), the roots of \( |1 - \alpha_1 z - \alpha_2 z^2| = 0 \) are outside the unit circle, and the sign of \( d \) is opposite to the sign of \( \lambda_1 \). The values of \( \lambda_1 \) and \( d \) are consistent with adjustment cost effects in production and are not compatible with the simple specification of the soil fertility deterioration in cotton production. In the Appendix I show that the interaction between land and fertilizer may affect the dynamics of the land allocations decision in such a way that if data on fertilizer are omitted \( \lambda_1 \) may be positive. Thus, the traditional omitted-variable argument may explain the signs of \( \lambda_1 \) and \( d \), maintaining the hypothesis of soil fertility deterioration in cotton. Using the estimated parameters I can calculate the response of land allocations to a permanent or temporary change in prices. The long-run supply elasticity, that is, the percentage change in the mean of \( A_t \) divided by the percentage change in the mean of \( R_t \) is equal to \(-1/3\); and the immediate effect of an expected change in \( R_t \), (short-run elasticity) has a

---

\(^{18}\) The maximization was done using the \texttt{opt} algorithm from the \texttt{optim} package of Princeton University. The complicated nonlinear structure of the model implies no gain from writing the analytical first and second derivatives; hence, I used the derivative-free method. I maintained a 10-digit accuracy level and checked for other local maxima in "the neighborhood." I do not report the asymptotic standard errors of our estimators since the Hessian, at the maximum, was not negative definite. The computer program had been tested using a Monte Carlo experiment of the same model that I estimated.
TABLE 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>.081</td>
</tr>
<tr>
<td>$d$</td>
<td>-.008</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>.524</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>.250</td>
</tr>
<tr>
<td>$\rho$</td>
<td>.081</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>1.55103</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>3.79</td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>-719.13</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>.06</td>
</tr>
</tbody>
</table>

Note: The log likelihood is $L(\theta) = -529.63$; $\beta$ = discount factor = .95, imposed a priori.

A lower value ($d < 0$) that is equal to $-11$. As such, all the estimated parameters of the model have signs consistent with the assumptions of the model, and the magnitudes of the long-run and the short-run elasticities are within a reasonable range.

Let $L(\theta)$ be the value of the log likelihood of the model and $L_\alpha$ be the value of the log likelihood of an estimated unrestricted version of the VAR (14). Then, $-2[L(\theta) - L_\alpha]$ is distributed $\chi^2(q)$, where $q$ is the number of restrictions that are tested. Three likelihood ratio tests of the model were performed. First, the null hypothesis that the parameters $\lambda_1$, $d$, $\alpha_1$, $\alpha_2$, and $\rho$ (not the constants and the dummies) are zero is tested against the model. (This test is similar to the standard $F$-test of the linear regression equation.) The likelihood value of the null hypothesis is $-529.63$, and it is rejected with a marginal significance level that is less than .001. Hence, the implications of the model on the pattern of serial and cross correlations between prices and area are more consistent with the data than the null hypothesis of no serial and cross correlations.

Second, I test the overidentifying cross-equations restrictions due to the rational expectations solution of the decision rule, maintaining the assumption that $A_1$ does not Granger cause $R$; that is, the zero restrictions on the three-lag VAR are not relaxed. The estimated VAR implied by the model is presented at the top of table 2. The estimated VAR where the cross-equation restrictions are not imposed is reported in the middle of table 2: the likelihood value of this system is $-505.489$. The marginal confidence level of testing the theory against the less restricted model is less than .5 ($\chi^2(2) = 1.2$), and the model is not rejected. Table 2 indicates that the coefficients of the restricted and unrestricted (with zero restrictions) VARs have the same sign as the auto- and cross covariances. The land equation shows a higher negative effect in the second-order lags of $A_1$ and $R$, and the

The average of $R = 16.8$. The average of $A_1 = 1,530.0$. The long-run elasticity is

$$\frac{\lambda_1}{d(1 - \lambda_1 \beta) (1 - \lambda_1)} = \frac{16.8}{1,530} = -.13.$$ 

The immediate elasticity (short run) = $(\lambda_1 / d)(16.8 / 1,530) = -.11$. 

19 The average of $R = 16.8$. The average of $A_1 = 1,530.0$. The long-run elasticity is
### Table 2

**The Estimates of the VAR (14)**

**Model's VAR, \(L(0) = -506.088\):**

\[
\begin{bmatrix}
A_{t+1} \\
R_t
\end{bmatrix} = \begin{bmatrix}
1,551.0 \\
3.8
\end{bmatrix} + \begin{bmatrix}
-719.1 \\
.66
\end{bmatrix} \text{WAR} + \begin{bmatrix}
.16 & 5.7 \\
0 & .52
\end{bmatrix} \begin{bmatrix}
A_{t-1} \\
R_{t-1}
\end{bmatrix} + \begin{bmatrix}
-.006 & -2.1 \\
0 & .25
\end{bmatrix} \begin{bmatrix}
A_{t-2} \\
R_{t-2}
\end{bmatrix} + \begin{bmatrix}
0 & -2.1 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
A_{t-3} \\
R_{t-3}
\end{bmatrix}
\]

**Unrestricted VAR (with zero restrictions), \(L_{u1} = -505.489\):**

\[
\begin{bmatrix}
A_{t+1} \\
R_t
\end{bmatrix} = \begin{bmatrix}
1,563.5 \\
3.9
\end{bmatrix} + \begin{bmatrix}
-724.5 \\
.81
\end{bmatrix} \text{WAR} + \begin{bmatrix}
.19 & -3.8 \\
0 & .56
\end{bmatrix} \begin{bmatrix}
A_{t-1} \\
R_{t-1}
\end{bmatrix} + \begin{bmatrix}
-.06 & -7.2 \\
0 & .21
\end{bmatrix} \begin{bmatrix}
A_{t-2} \\
R_{t-2}
\end{bmatrix} + \begin{bmatrix}
0 & -4.4 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
A_{t-3} \\
R_{t-3}
\end{bmatrix}
\]

**Unrestricted VAR (without zero restrictions), \(L_{u2} = -495.9\):**

\[
\begin{bmatrix}
A_{t+1} \\
R_t
\end{bmatrix} = \begin{bmatrix}
1,197.6 \\
-14.2
\end{bmatrix} + \begin{bmatrix}
-661.4 \\
4.6
\end{bmatrix} \text{WAR} + \begin{bmatrix}
.27 & -7.6 \\
.005 & .86
\end{bmatrix} \begin{bmatrix}
A_{t-1} \\
R_{t-1}
\end{bmatrix} + \begin{bmatrix}
-.11 & -8.7 \\
.002 & .11
\end{bmatrix} \begin{bmatrix}
A_{t-2} \\
R_{t-2}
\end{bmatrix} + \begin{bmatrix}
.21 & 9.7 \\
.004 & .36
\end{bmatrix} \begin{bmatrix}
A_{t-3} \\
R_{t-3}
\end{bmatrix}
\]
patterns of the moving average coefficients of the two VARs are almost identical. This test can be viewed as a support for the rational expectations solution versus any econometric model that ignores the cross-equation restrictions but that is observationally equivalent (Sargent 1976) to the model presented here. The Nerlovian supply response model is an example of an econometric model that for some specifications has the same VAR as for the model presented here.\(^{20}\)

Third, the likelihood ratio test of the model versus a three-lag unrestricted VAR (the third system in Table 2) rejects the null hypothesis with a marginal confidence level of .995 (\(\chi^2[7] = 20.3\)). The signs of the serial and cross correlations of the area equation are the same as in the model, but the magnitude is significantly different. The main change is in the \(R\) equation where the zero restrictions are relaxed. The serial correlations of \(R\) have the same sign and only marginally different values compared with the VAR of the model. Hence, the model is rejected because of the zero restrictions on the price equation and not because of the rational expectations cross-equations restrictions. The result indicates that the model probably fails because of the way I assume prices are determined in the Egyptian economy rather than because of the particular linear-quadratic approximation of the dynamic technology or the implied rational expectation restrictions. Given a priori acceptable point estimates and the positive outcome of the second test, I can characterize the empirical results as providing some support for the specific model.

IV. Conclusion

This work is best viewed as an attempt to construct a dynamic stochastic theory of agricultural supply that can be used to interpret observed data on land allocations, crop yields, and prices. By introducing an explicit approximation to a well-known characteristic of the crop production process (depletion of soil productivity), I demonstrate how the dynamic properties of land allocation and their interaction with crop prices depend on the production technology. The model can interpret the dynamics of land allocations as a result of different technologies: the depletion effect in land productivity, costs of adjusting crop areas, or omitted inputs that interact with land (e.g., fertilizer). In such a model the supply response elasticities are functions of the technology and the parameters of the price processes. It turns out that the restrictions imposed by the model received some support from the data. The estimated structural parameters are consistent with the model and conform to a cost-of-adjustment framework, and

\(^{20}\) See Askari and Cummings (1976) for reported results from the Nerlovian model.
the cross-equations restrictions are not rejected when the zero restrictions on the price equation are maintained. The methodology adopted here requires that future changes in the model demand careful consideration of aspects of the agricultural decision process that have been ignored previously. As such, there is hope that future research will lead us to an improved understanding of the dynamics of agricultural supply.

Appendix

Here I consider a simple example with fertilizer. Let $F_{i_t}$ be the fertilizer that is allocated to crop 1 at time $t$, and let the production function for crop 1 be

$$X_{1t} = \left( f_{1t} + \sigma_{1t} - \frac{g_{1t}}{2} A_{11} \right) A_{1}, + \delta \left[ w_{1} F_{1t} - \left( \frac{A_{11t-1}}{A} - \frac{A_{11t}}{A} \right) \right] A_{1} - \frac{w_{2}}{2} F_{1t}^{2}$$

(A1)

where $w_{1}$ and $w_{2}$ are positive scalars. Substituting (A1) rather than (5) into (3) and subtracting the cost of fertilizer from the farmer’s problem, I can find the first-order necessary conditions of the farmer’s problem with respect to $F_{1t}$ and $A_{1t}$. Solving for $F_{1t}$ in terms of $A_{1t}$ and the current price of fertilizer and substituting the result into the first-order condition with respect to $A_{1t}$, I find that the land allocation decision rule has exactly the same form as the solution for the original problem (7). Here the price of fertilizer is an additional element in the optimal decision rule and in the uncontrolled vector stochastic process of $Z$. However, there exists an important difference between the two solutions. If $d_{1}^{2} w_{1}/w_{2} > g_{1} + 2(d_{1}/A)$, the coefficient of $A_{11}$ in the Euler equation is negative and it is possible to have a real solution with $0 < \lambda_{1} < 1$. Hence the serial correlation in land allocation is positive, such as in the adjustment costs case. The intuitive interpretation of this result is that if the production of crop 1 is very responsive to fertilizer applications (large $w_{1}$ and small $w_{2}$), the rotation element in land allocation may completely disappear.

References


